$$f(x) = \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

$$\begin{tikzcd}
 & x \arrow[dr, "\tilde{f}_3"] & & & \\
x \arrow[ur, "\tilde{f}_1"] \arrow[dr, "\tilde{f}_2"'] & & x(x+1) \arrow[r, "\tilde{f}_4"] & \frac{1}{x(x+1)} \\
 & x+1 \arrow[ur] & & &
\end{tikzcd}$$

$$\tilde{f}_1(x) = x(1+\varepsilon_1)$$

$$\tilde{f}_2(x) = (x(1+\varepsilon_1)+1)(1+\varepsilon_2) \lessapprox x(1+2\varepsilon_3)+(1+\varepsilon_2)$$

$$\tilde{f}_3(x) = (x(1+\varepsilon_1))(x(1+2\varepsilon_3)+1+\varepsilon_2)(1+\varepsilon_4)$$

$$\lessapprox (x^2(1+3\varepsilon_5)+x(1+2\varepsilon_6))(1+\varepsilon_4)$$

$$\lessapprox x^2(1+4\varepsilon_7)+x(1+3\varepsilon_8)$$

$$\tilde{f}_4 = \frac{(1\pm\varepsilon_9)}{x^2(1\pm 4\varepsilon_7)+x(1\pm 3\varepsilon_8)} \lessapprox \frac{1\pm\varepsilon_9}{x^2+x+\varepsilon_{10}(4x^2+3x)}$$

$$\lessapprox \frac{1\pm\varepsilon_9}{(x^2+x)\cdot\left(1\pm\varepsilon_{10}\frac{4x^2+3x}{x^2+x}\right)} \lessapprox \frac{1\pm\varepsilon_9}{x^2+x}\left(1\pm\varepsilon_{10}\frac{4x^2+3x}{x^2+x}\right)$$

$\lessapprox 4$ pour x grand

$$\lessapprox \frac{1}{x^2+x}(1\pm 5\varepsilon_{11})$$

$$\left|\frac{\tilde{f}(x)-f(x)}{f(x)}\right| \lessapprox \left|\frac{\frac{1}{x^2+x}(1\pm 5\varepsilon_{11}) - \frac{1}{x^2+x}}{\frac{1}{x^2+x}}\right| \leqslant 5\varepsilon_m + O(\varepsilon_m)$$

donc forward stable. (la constante ne dépend pas de x)

Pour $\tilde{g}$

$$
\begin{tikzcd}
 & & \frac{1}{x} \arrow[ddrr, bend left] & & \\
x \arrow[r, "fl"] & x \arrow[ur, "\tilde{g}_1"] \arrow[dr, "\tilde{g}_2"'] & & & \\
 & & x+1 \arrow[r, "\tilde{g}_3"'] & \frac{1}{x+1} \arrow[r, "\tilde{g}_4"'] & \frac{1}{x}-\frac{1}{x+1}
\end{tikzcd}
$$

$$fl(x) = x(1 \pm \varepsilon_1)$$

$$\tilde{g}_1(x) = \frac{1}{x(1\pm\varepsilon_1)}(1\pm\varepsilon_2) \approx \frac{1}{x}(1\pm 2\varepsilon_3)$$

$$\tilde{g}_2(x) = (x(1\pm\varepsilon_1)+1)(1\pm\varepsilon_3) \approx x(1\pm 2\varepsilon_4)+(1\pm\varepsilon_3)$$

$$\tilde{g}_3(x) = \frac{1\pm\varepsilon_5}{x(1\pm 2\varepsilon_4)+(1\pm\varepsilon_3)} \approx \frac{1\pm\varepsilon_5}{x+1\pm\varepsilon_6(2x+1)} \approx \frac{1\pm\varepsilon_5}{(x+1)\left(1\pm\varepsilon_6\frac{2x+1}{x+1}\right)}$$

$$\approx \frac{1\pm\varepsilon_5}{x+1}\left(1\pm\varepsilon_6 \underbrace{\frac{2x+1}{x+1}}_{\approx 2 \text{ pour } x \text{ grand}}\right) \approx \frac{1}{x+1}(1\pm 3\varepsilon_7)$$

$$\tilde{g}_4(x) \approx \left(\frac{1}{x}(1\pm 2\varepsilon_3) - \frac{1}{x+1}(1\pm 3\varepsilon_7)\right)(1\pm\varepsilon_8)$$

$$\approx \frac{1}{x}(1\pm 3\varepsilon_9) - \frac{1}{x+1}(1\pm 4\varepsilon_{10}) \approx \frac{1}{x}-\frac{1}{x+1} \pm \frac{3}{x}\varepsilon_9 \pm \frac{4\varepsilon_{10}}{x+1}$$

$$\approx \frac{1}{x}-\frac{1}{x+1} \pm \frac{3\varepsilon_9(x+1)+4x\varepsilon_{10}}{x(x+1)} \approx \left(\frac{1}{x}-\frac{1}{x+1}\right)\cdot\left(1\pm\varepsilon_{11}(7x+3)\right)$$

$$\left|\frac{\tilde{g}(x)-g(x)}{g(x)}\right| \approx \left|\frac{\cancel{\left(\frac{1}{x}-\frac{1}{x+1}\right)}(\cancel{1}\pm\varepsilon_{11}(7x+3)) - \cancel{\left(\frac{1}{x}-\frac{1}{x+1}\right)}}{\cancel{\left(1/x - 1/x+1\right)}}\right|$$

$$\lesssim 3\varepsilon_m + \underbrace{7x}_{\text{grand pour } x \text{ grand !}}\varepsilon_m + O(\varepsilon_m)$$

donc $\tilde{g}$ n'est pas forward stable.