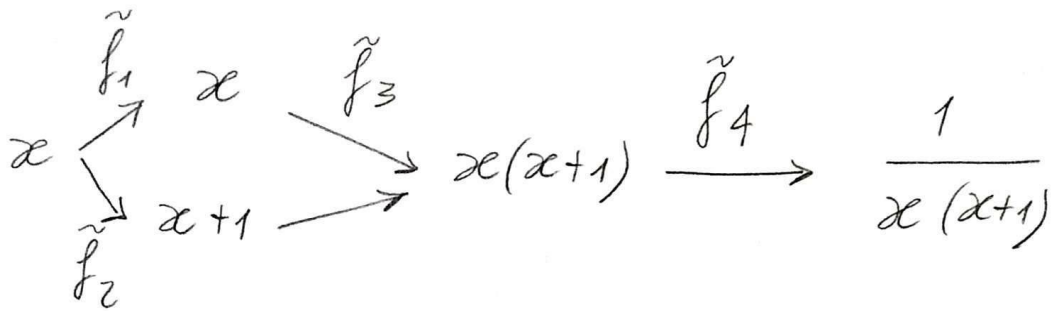


$$f(x) = \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$



$$\tilde{f}_1(x) = x(1 + \varepsilon_1)$$

$$\tilde{f}_2(x) = (x(1 + \varepsilon_1) + 1)(1 + \varepsilon_2) \approx x(1 + 2\varepsilon_3) + (1 + \varepsilon_2)$$

$$\begin{aligned} \tilde{f}_3(x) &= (x(1 + \varepsilon_1))(x(1 + 2\varepsilon_3) + 1 + \varepsilon_2)(1 + \varepsilon_4) \\ &\approx (x^2(1 + 3\varepsilon_5) + x(1 + 2\varepsilon_6))(1 + \varepsilon_4) \\ &\approx x^2(1 + 4\varepsilon_7) + x(1 + 3\varepsilon_8) \end{aligned}$$

$$\tilde{f}_4 = \frac{(1 + \varepsilon_9)}{x^2(1 + 4\varepsilon_7) + x(1 + 3\varepsilon_8)} \approx \frac{1 \pm \varepsilon_9}{x^2 + x + \varepsilon_{10}(4x^2 + 3x)}$$

$$\approx \frac{1 \pm \varepsilon_9}{(x^2 + x) \cdot (1 \pm \varepsilon_{10} \frac{4x^2 + 3x}{x^2 + x})} \approx \frac{1 \pm \varepsilon_9}{x^2 + x} \left(1 \pm \varepsilon_{10} \frac{4x^2 + 3x}{x^2 + x}\right)$$

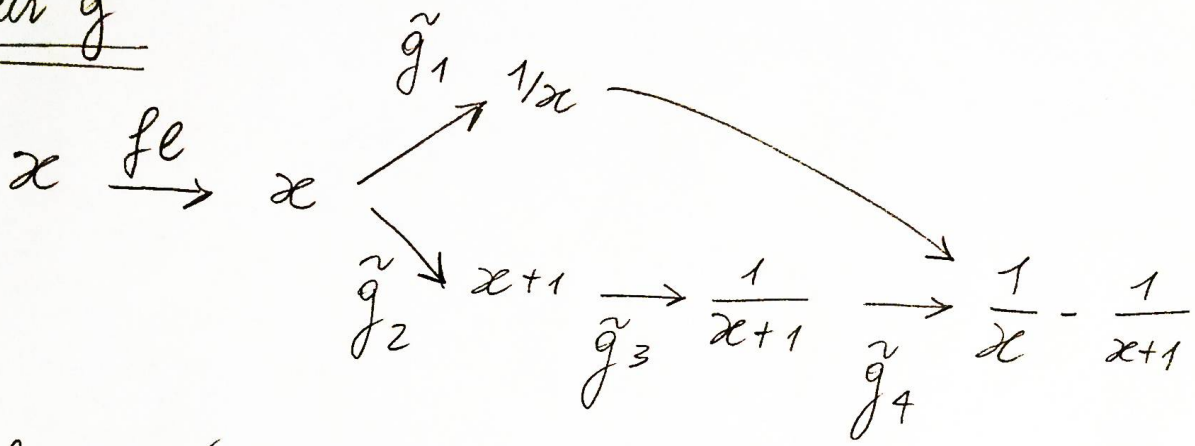
$\approx 4$   
power  $x$   
 $\nearrow$  grand

$$\approx \frac{1}{x^2 + x} (1 \pm 5\varepsilon_{11})$$

$$\left| \frac{\tilde{f}(x) - f(x)}{f(x)} \right| \approx \left| \frac{\frac{1}{x^2 + x} (1 \pm 5\varepsilon_{11}) - \frac{1}{x^2 + x}}{\frac{1}{x^2 + x}} \right| \leq 5\varepsilon_{11} + o(\varepsilon_{11})$$

donc forward stable. (la constante ne dépend pas de  $x$ )

Pour  $\tilde{g}$



$$fl(x) = x(1 \pm \varepsilon_1)$$

$$\tilde{g}_1(x) = \frac{1}{x(1 \pm \varepsilon_1)} (1 \pm \varepsilon_2) \approx \frac{1}{x} (1 \pm 2\varepsilon_3)$$

$$\tilde{g}_2(x) = (x(1 \pm \varepsilon_1) + 1) (1 \pm \varepsilon_3) \approx x(1 \pm 2\varepsilon_4) + (1 \pm \varepsilon_3)$$

$$\tilde{g}_3(x) = \frac{1 \pm \varepsilon_5}{x(1 \pm 2\varepsilon_4) + (1 \pm \varepsilon_3)} \approx \frac{1 \pm \varepsilon_5}{x+1 \pm \varepsilon_6(2x+1)} \approx \frac{1 \pm \varepsilon_5}{(x+1)(1 \pm \varepsilon_6 \frac{2x+1}{x+1})}$$

$$\approx \frac{1 \pm \varepsilon_5}{x+1} (1 \pm \varepsilon_6 \frac{2x+1}{x+1}) \approx \frac{1}{x+1} (1 \pm 3\varepsilon_7)$$

$\approx 2$  pour  $x$  grand

$$\tilde{g}_4(x) \approx \left( \frac{1}{x} (1 \pm 2\varepsilon_3) - \frac{1}{x+1} (1 \pm 3\varepsilon_7) \right) (1 \pm \varepsilon_8)$$

$$\approx \frac{1}{x} (1 \pm 3\varepsilon_9) - \frac{1}{x+1} (1 \pm 4\varepsilon_{10}) \approx \frac{1}{x} - \frac{1}{x+1} \pm \frac{3}{x} \varepsilon_9 \pm \frac{4}{x+1} \varepsilon_{10}$$

$$\approx \frac{1}{x} - \frac{1}{x+1} \pm \frac{3\varepsilon_9(x+1) + 4x\varepsilon_{10}}{x(x+1)} \approx \left( \frac{1}{x} - \frac{1}{x+1} \right) \cdot (1 \pm \varepsilon_{11} (7x+3))$$

$$\left| \frac{\tilde{g}(x) - g(x)}{g(x)} \right| \approx \left| \frac{\left( \frac{1}{x} - \frac{1}{x+1} \right) (1 \pm \varepsilon_{11} (7x+3)) - \left( \frac{1}{x} - \frac{1}{x+1} \right)}{\left( \frac{1}{x} - \frac{1}{x+1} \right)} \right|$$

$$\leq 3\varepsilon_{11} + 7x\varepsilon_{11} + o(\varepsilon_{11})$$

grand pour  $x$  grand!

donc  $\tilde{g}$  n'est pas forward stable.